DOCUMENT RESUME

ED 348 377 TM 018 739

AUTHOR Li, Jianmin; And Others
TITLE Multiple Testing with Modified Bonferroni Methods.
PUB DATE Apr 92
NOTE 29p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 20-24, 1992).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Comparative Analysis; *Correlation; *Hypothesis Testing; *Mathematical Models; Matrices; Research Methodology; Statistical Distributions
IDENTIFIERS *Bonferroni Procedure; Multiple Comparisons; *Multiple Testing; P Values; Type I Errors

ABSTRACT

This paper discusses the issue of multiple testing and overall Type I error rates in contexts other than multiple comparisons of means. It demonstrates, using a 5 x 5 correlation matrix, the application of 5 recently developed modified Bonferroni procedures developed by the following authors: (1) Y. Hochberg (1988); (2) B. S. Holland and M. D. Copenhaver (1987); (3) S. Holm (1979); (4) G. Hommel (1988); and (5) D. M. Rom (1990). The original Bonferroni procedure is compared with these modified procedures, using 50 correlation matrices reported in educational and psychological journals. A total of 2,587 p-values were obtained. Using an overall alpha per matrix of 0.05, 0.10, and 0.20, the number of rejections were counted. For a relatively large number of matrices, the modified Bonferroni procedures rejected the same number of hypotheses as did the original Bonferroni procedure. However, whenever there were differences, all Bonferroni procedures rejected more hypotheses than did the original procedure. Among the four modified procedures, the Holm procedure had the lowest and the Hommel procedure had the highest number of rejections. The larger the Type I error rate, the greater was the difference among the methods in the number of rejections. The differences in the number of rejections may depend on the distribution of a set of p-values. Thirteen tables contain the analysis results, and there is a 39-item list of references. (Author/SLD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED348377

MULTIPLE TESTING WITH MODIFIED BONFERRONI METHODS

Jianmin Li

Stephen Olejnik

Carl J Huberty

University of Georgia

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

JIANMIN LI

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Paper presented at the annual meeting of the American Educational Research Association, San Francisco, April 1992

TM018736

BEST COPY AVAILABLE

ABSTRACT

The present paper a) Discusses the issue of multiple testing and overall Type I error rates in contexts other than multiple comparisons of means; b) Demonstrates, using a 5x5 correlation matrix, the application of five recently developed modified Bonferroni procedures [Hochberg (1988), Holland and Copenhaver (1987), Holm (1979), Hommel (1988), and Rom (1990)]; and c) Compares the original Bonferroni procedure with four modified Bonferroni procedures, using 50 correlation matrices reported in educational and psychological journals. A total of 2587 p-values were obtained. Using an overall α per matrix of .05, .10, and .20, the number of rejections were counted. For a relatively large number of matrices, the modified Bonferroni procedures rejected the same number of hypotheses as the original Bonferroni procedure. However, whenever there were differences: a) All modified Bonferroni procedures rejected more hypotheses than the original Bonferroni procedure; b) Among the four modified procedures, the Holm procedure had the lowest number of rejections and the Hommel procedure had the highest number of rejections; c) The larger the overall Type I error rate the greater the difference among the methods in the number of rejections; and d) The differences in the number of rejections may depend on the distribution of a set of p-values.

Introduction

There have been several discussions on the issue of controlling the overall Type I error rate in situations where multiple tests are conducted simultaneously. Ryan (1959) discussed three types of Type I error rates: error rate per comparison, error rate per experiment, and error rate experimentwise. Similar concepts have been discussed recently by book authors (Keppel, 1991; Keppel & Zedeck, 1989; Maxwell & Delaney, 1990; Toothaker, 1991). In general, Type I error rate can be controlled at two levels: (1) Type I error rate per individual hypothesis test, and (2) overall Type I error rate for a set of hypotheses tested.

In the context of multiple comparisons of means, the necessity of controlling overall Type I error rate has been widely accepted, although there are different points of view (Holland, 1991; Lea, 1991; Saville, 1990). Many multiple comparison procedures have been developed and applied. Studies for comparing these methods have been conducted (Dunnett & Tamhane, 1992; Hsiung & Olejnik, 1991; Keselman, Keselman, & Games, 1991; Keselman, Keselman, & Shaffer, 1991; Seaman, Levin, & Serlin, 1991; Seaman, Levin, Serlin, & Franke 1990). However, the same level of attention has not been given to other multiple test situations, such as a matrix of correlations, factorial ANOVA, multiple regression, multiple contingency tables, multiple experiments, and multiple outcome variable studies.

The Bonferroni approach controls overall Type I error by dividing the selected overall alpha level by the total number of tests. This is a commonly used technique and can be easily applied in any multiple test situation. A major problem with the original Bonferroni approach is that it lowers the statistical power of detecting true false hypotheses. Several suggestions for modifying the original Bonferroni procedure have been made by statisticians aimed at increasing statistical power. Five of these alternatives were reviewed by Holland and Copenhaver (1988). More recently, Hochberg (1988), Hommel (1988), and Rom (1990) have proposed additional modified Bonferroni procedures. In exchange for higher statistical power, the newer approaches are more complex.

The purpose of the present study is three fold: (1) Discuss the issue of multiple testing and overall Type I error rate in contexts other than multiple comparisons of means; (2) Demonstrate the application of five recently developed modified Bonferroni procedures; and (3) Compare the original Bonferroni procedure with four modified Bonferroni procedures in the number of rejections, using 50 correlation matrices reported in educational and psychological journals.

Multiple Tests and Controlling Overall Type I Error Rate

Ryan (1959) discussed five situations where multiple testing can be applied: (1) multiple comparisons, (2) multiple tests with intercorrelated variables, (3) factorial design,

(4) replicated tests of a single hypothesis, and (5) overlapping measures relating to a single hypothesis. Stevens (1986) also discussed five common situations other than one way ANOVA, where multiple statistical tests should be recognized: (1) factorial ANOVA, (2) factorial ANOVA with multiple dependent variables, (3) multiple chi-squared tests in a survey research, (4) examining the elements of a correlation matrix for significance, and (5) comparing the results of many studies in a given content area.

Several researchers have discussed or commended on the issue of Type I error rate in contexts other than multiple comparisons of means: independent group factorial ANOVA (Fletcher, Daw, & Young, 1989), correlation matrix (Collis & Rosenblood, 1985; Larzelere & Mulaik, 1977), multiple F tests with multiple outcome variables (Manly & McAlevey, 1987), and factorial mixed model design (Looney & Stanley, 1989). All these studies raised the issue of controlling overall Type I error rate, and suggested a corresponding solution for the particular situation.

In several recently published educational research textbooks, authors have discussed the issue of overall Type I error rate and methods of controlling it for a variety of multiple test situations. Keppel (1991, p. 247-249) emphasized awareness of familywise Type I error for simple effects tests. Stevens (1990, p. 135-137) introduced the Holm (1979) procedure as a alternative to the original Bonferroni procedure. Maxwell and Delaney (1990) discussed an adjusted criterion alpha level in an analysis of covariance context as well as in a within-subject design context with both multivariate and univariate approaches. Darlington (1990) devoted an entire chapter (Chapter 11) to discussing multiple tests and methods. It appears that the original Bonferroni procedure is the most commonly recommended method of controlling overall Type I error rate.

A number of modifications to the original Bonferroni procedure have been developed since the late 1970s by Hochberg (1988), Holland and Copenhaver (1987), Holm (1979), Hommel (1988), Shaffer (1986), Simes (1986), Rom (1990), and Worsley (1982). Some graphical procedures for multiple tests have also been investigated (Barton & Huberty, 1989; Hill, 1969; Hochberg & Benjamini, 1990; Koziol, 1989;).

However, few of the new techniques have been introduced in educational research and statistics textbooks. Also, the comparisons across newly developed methods are quite limited, especially when using real data. It is our hope that the work of this paper will increase reader awareness of these new statistical methods and their practical usefulness in educational research settings.

Applications of Five Modified Bonferroni Procedures

This section demonstrates the numerical calculations for the original Bonferroni procedure and five modified Bonferroni procedures: Hochberg (1988), Holland and Copenhaver (1987), Holm (1979), Hommel (1988), and Rom (1990). Dunnett and Tamhane (1992) categorized these procedures in three groups: single-step (SS), step-down (SD), and step-up (SU). These abbreviations are used for the current study. The SS procedure (original Bonferroni) sets a single criterion for testing all individual hypotheses. The SD

and SU procedures order the hypotheses to be tested by their p values, and then compute adjusted significant levels for each individual hypothesis. The SD (Holm, Holland-Copenhaver) procedures start the testing with the hypothesis with the smallest p value, whereas the SU procedures (Hochberg, Hommel, Rom) start the testing with the hypothesis with the largest p value.

A 5x5 correlation matrix with n=154 is used as an example for the purpose of demonstration (Richardson, 1988). Table 1 presents the correlation coefficients and corresponding p values. Table 2 lists the r values and corresponding p values in an ascending order by the value of p. The $H_{(i)}$ column indicates the order of the hypotheses.

Insert Table 1 and 2 about here

For notation consistency across all procedures, we use α to denote the overall Type I error rate per matrix, α' to denote a criterion for testing an individual hypothesis, i to indicate the order of the hypotheses, and m as the total number of hypotheses tested. For all procedures demonstrated in this section, $\alpha=.10$ and $m=10$.

Original Bonferroni Procedure (SS)

The original Bonferroni procedure computes $\alpha'=\alpha/m$. For our example, the adjusted significance level for rejecting each individual hypotheses is $\alpha'=.10/10=.01$. The hypotheses $H_{(1)}$, $H_{(2)}$, $H_{(3)}$, and $H_{(4)}$ have $p<.01$, so they are rejected. The remaining hypotheses, $H_{(5)}$ to $H_{(10)}$, are retained.

Holm Procedure (SD)

Holm (1979) proposed sequentially setting different significance levels for rejecting each individual hypothesis: let $p_{(1)},\ldots,p_{(m)}$ be the ordered p values and $H_{(1)},\ldots,H_{(m)}$ be the corresponding hypotheses. Suppose i is the smallest integer from 1 to m such that

$$p_{(i)} > \alpha/(m-i+1),$$

the Holm procedure rejects $H_{(1)}$ to $H_{(i-1)}$ and retains $H_{(i)}$ to $H_{(m)}$. In our example, $\alpha_{(i)}'=.10/(10-i+1)$.

1. For $H_{(1)}$, $\alpha_{(1)}'=.10/(10-1+1)=.01$. Comparing the obtained $p_{(1)}$ value of .0000 with $\alpha_{(1)}'$, $p_{(1)}<.01$.

2. For $H_{(2)}$, $\alpha_{(2)}'=.10/(10-2+1)=.0111$. Comparing the obtained $p_{(2)}$ value of .0001 with $\alpha_{(2)}'$, $p_{(2)}<.0111$.

The testing continues until the sixth hypothesis where $\alpha_{(6)}'=.10/(10-6+1)=.0200$. The obtained $p_{(6)}$ is .0255, $p_{(6)}>\alpha_{(6)}'$. The hypothesis testing stops. The hypotheses $H_{(1)}$ to

$H_{(5)}$ are rejected, and the hypotheses from $H_{(6)}$ to $H_{(10)}$ are retained. Table 3 presents the step by step testing.

Insert Table 3 about here

Instead of evenly splitting the overall α level among all the hypotheses tested, the Holm procedure computes $\alpha_{(i)}'$ for each individual $H_{(i)}$ starting from the hypothesis with the smallest p value ($H_{(1)}$) until $p_{(i)} > \alpha_{(i)}'$. Then rejecting $H_{(1)}$ to $H_{(i-1)}$, and retaining $H_{(i)}$ to $H_{(m)}$ without further calculation.

Holland and Copenhaver procedure (SD)

The Holland and Copenhaver (1987) procedure may be described as follows: let $p_{(1)},\ldots,p_{(m)}$ be the ordered p values and $H_{(1)},\ldots,H_{(m)}$ be the corresponding hypotheses. Suppose i is the smallest integer from 1 to m such that

$$p_{(i)} > 1-(1-\alpha)^{1/(m-i+1)},$$

the Holland and Copenhaver procedure rejects $H_{(1)}$ to $H_{(i-1)}$ and retains $H_{(i)}$ to $H_{(m)}$. In our example, $\alpha_{(i)}'=1-(1-.10)^{1/(10-i+1)}$.

1. For $H_{(1)}$, $\alpha_{(1)}'=1-(1-.10)^{1/(10-1+1)}=.0105$. Comparing the obtained $p_{(1)}$ value of .0000 with $\alpha_{(1)}'$, $p_{(1)}<.0105$.

2. For $H_{(2)}$, $\alpha_{(2)}'=1-(1-.10)^{1/(10-2+1)}=.0116$. Comparing the obtained $p_{(2)}$ value of .0001 with $\alpha_{(2)}'$, $p_{(2)}<.0116$.

The testing continues until the sixth hypothesis where $\alpha_{(6)}'=1-(1-.05)^{1/(10-6+1)}=.0209$. The obtained $p_{(6)}$ is .0255, $p_{(6)}>\alpha_{(6)}'$. The testing stops. Thus $H_{(1)}$ to $H_{(5)}$ are rejected, and all the hypotheses from $H_{(6)}$ to $H_{(10)}$ are retained. Table 4 presents the step by step testing.

Insert Table 4 about here

Like the Holm procedure, the Holland and Copenhaver procedure computes $\alpha_{(i)}'$ for each $H_{(i)}$ starting from the hypothesis with the smallest p value until $p_{(i)} > \alpha_{(i)}'$; then rejecting $H_{(1)}$ to $H_{(i-1)}$, and retaining $H_{(i)}$ to $H_{(m)}$ without further calculation.

Hochberg Procedure (SU)

Hochberg (1988) developed the first step-up approach: for any i=m, m-1,...,1, if

$$p_{(i)} \leq \alpha/(m-i+1),$$

then reject $H_{(1)}$ to $H_{(i)}$, and retain $H_{(i+1)}$ to $H_{(m)}$. Numerical calculations for this procedure are the same as for the Holm procedure. The only difference is that this procedure starts testing the hypothesis with the largest p value. For our example, $\alpha_{(i)}'=.10/(10-i+1)$ starts at i=10.

1. For $H_{(10)}$, $\alpha_{(10)}'=.10/(10-10+1)=.10$. Comparing the obtained $p_{(10)}$ value of .2670 with $\alpha_{(10)}$, $p_{(10)}>.10$.

2. For $H_{(9)}$, $\alpha_{(9)}'=.10/(10-9+1)=.05$. Comparing the obtained $p_{(9)}$ value of .0633 with $\alpha_{(9)}'$, $p_{(9)}>.05$.

The testing continues until $H_{(5)}$, where $\alpha_{(5)}'=.10/(10-5+1)=.0167$. The obtained $p_{(5)}$ value is .0129, $p_{(5)}<\alpha_{(5)}'$. The testing stops at $H_{(5)}$ without further calculation. Hypotheses $H_{(10)}$ to $H_{(6)}$ are retained, while $H_{(5)}$ to $H_{(1)}$ are rejected. The Hochberg procedure is very similar to the Holm procedure. The only difference is that it takes a step-up (SU) approach. Table 5 presents the step by step testing.

Insert Table 5 about here

Hommel Procedure (SU)

Hommel (1988) proposed another step-up approach. First, let $J=\{i'\in\{1,...,m\}: p_{(m-i'+k)} > k\alpha/i'; k=1,...,i'\}$. Then, if J is nonempty, reject $H_{(i)}$ whenever $p_{(i)}\leq\alpha/j'$ with $j'=\max J$. If J is empty, reject all $H_{(i)}(i=1,...,m)$.

This procedure includes two stages of decision making. The first stage uses the obtained p values to compute the number of members in J. The second stage obtains the significance level of rejection using $\alpha'=\alpha/j'$, where j' is the largest number in J. The uniqueness of the Hommel procedure is that it not only considers the order of the tests but also takes the obtained p values into the calculation while computing the α'.

The first stage is further broken down to steps (denoted as i'), and within each step, one or more tests (denoted as k) are conducted. At the first step i'=1, and it increases 1 at each step. k=1 to i' within each step.

Step 1 only contains one test. With m=10, i'=1, and k=1, $p_{(m-i'+k)}=p_{(10-1+1)}=p_{(10)}$ =.2670. The criterion value is $k\alpha/i'=1(.10)/1=.10$. Since $p_{(10)}>.10$, the procedure continues with j=1.

Step 2 contains two tests. For the first test in step 2, i'=2, k=1, $p_{(m-i'+k)}=p_{(10-2+1)}$ $=p_{(9)}=.0633$, and $k\alpha/i'=1(.10)/2=.05$. Because $p_{(9)}>.05$, the procedure continues. For the second test in step 2, i'=2, k=2, $p_{(m-i'+k)}=p_{(10-2+2)}=p_{(10)}=.2670$, and $k\alpha/i'=2(.10)/2=.10$, with $p_{(10)}>.10$. Because at step 2 both $p_{(9)}$ and $p_{(10)}$ are larger than the corresponding criterion values, the procedure continues with j=2.

Step 3 contains three tests. For the first test in step 3, i'=3, k=1, $p_{(m-i'+k)}=p_{(10-3+1)}=p_{(8)}=.0633$, and $k\alpha/i'=1(.10)/3=.0333$. Because $p_{(8)}>.0333$, the procedure continues. For the second test in step 3, i'=3, k=2, and $p_{(m-i'+k)}=p_{(10-3+2)}=p_{(9)}=.0633$. The criterion value is $k\alpha/i'=2(.10)/3=.0667$. Because $p_{(9)}<.0667$, the procedure at the first stage stops. The largest number in J is 2, so j'=2. If $p_{(9)}>.0667$, the procedure would continue to $p_{(10)}$. If $p_{(10)}$ is also larger than $k\alpha/i'=3(.10)/3=.10$, we would continue to step 4 of testing. At this point, j would equal to 3.

In general, for each test, we select obtained the p value $p_{(m-i'+k)}$ to compare with a computed criterion value $k\alpha/i'$. If $p_{(m-i'+k)}>k\alpha/i'$, the calculations move on to the next $p_{(m-i'+k)}$. If we have $p_{(m-i'+k)}>k\alpha/i'$ for all the tests within a step, we increase j by 1; otherwise the calculations stop. Table 6 demonstrates the calculations and decision making process in the first stage.

Insert Table 6 about here

For the second stage of the Hommel procedure, reject $H_{(i)}$ whenever $p_{(i)}\leq\alpha/j'$. In our example, j'=2, $\alpha/j'=.10/2=.05$. Therefore, hypotheses $H_{(1)}$ to $H_{(7)}$ are rejected. If $p_{(10)}<.10$ at step 1, we would reject all the hypotheses.

Rom Procedure (SU)

Rom (1990) developed a even more complicated procedure. With this procedure, it is more convenient to denote $H_{(1)}$ as the hypothesis with the largest p value and $H_{(m)}$ as the hypothesis with the smallest p value.

The testing starts by comparing $p_{(1)}$ with $\alpha_{(1)}'$ and stops when $p_{(i)}<\alpha_{(i)}'$. Then $H_{(1)}$ to $H_{(i+1)}$ are retained and $H_{(i)}$ to $H_{(m)}$ are rejected. The computing equation for solving α_i's can be divided into three parts. The first part is $\alpha^1+\alpha^2+...+\alpha^{i-1}$ and the second part is $\binom{i}{1}(\alpha_{(2)}{}'^{i-1})+\binom{i}{2}(\alpha_{(3)}{}'^{i-2})+...+\binom{i}{i-2}(\alpha_{(i-1)}{}'^{2})$. To solve for α_i', subtract the second part from the first part, and divide the difference by i.

Step 1. $\alpha_{(1)}'$ is always equal to overall α. In our example, $\alpha_{(1)}'=.10$.

Step 2. $\alpha_{(2)}'$ is always equal to $\alpha_{(1)}'/2$. So, $\alpha_{(2)}'=.05$.

Step 3. $\alpha_{(3)}'=[\alpha+\alpha^2-3(\alpha_{(2)}{}'^2)]/3=.0342$. The 3 before the $(\alpha_{(2)}{}'^2)$ is the result of $\binom{3}{1}$, which is the number of combinations of 3 things taking 1 out at a time.

Step 4. $\alpha_{(4)}'=[\alpha+\alpha^2+\alpha^3-4(\alpha_{(2)}{}'^3)-6(\alpha_{(3)}{}'^2)]/4=.0259$. The 4 before the $(\alpha_{(2)}{}'^3)$ is the result of $\binom{4}{1}$, and 6 before the $(\alpha_{(3)}{}'^2)$ is the result of $\binom{4}{2}$.

When testing $H_{(6)}$, $p_{(6)} < \alpha_{(6)}$'. Therefore, $H_{(6)}$ and all the hypotheses with p smaller than $p_{(6)}$ are rejected, and rest of the hypotheses are retained. The procedure thus stops. The same algorithm is used for computing any of the $\alpha_{(i)}$'s. However, the computation procedure becomes more and more complicated when i increases. Table 7 demonstrates the numerical calculations for computing $\alpha_{(1)}$' to $\alpha_{(6)}$'.

Insert Table 7 about here

Number of Hypotheses Rejected by Each Procedure

The original Bonferroni procedure and the five modified Bonferroni procedures are applied to our example matrix. A comparison in the number of rejections was made for overall α levels at .05, .10, and .20 (see Table 8). When α=.05, there was no differences in the number of rejections across all the procedures. When α=.10, compared with the original Bonferroni, the Hommel procedure rejected three more hypotheses and the remaining modified procedures rejected one more hypothesis. When α=.20, all the modified procedures rejected four more hypotheses than the original Bonferroni procedure.

Insert Table 8 about here

Empirical Comparisons among the Procedures

All modified Bonferroni procedures have been mathematically shown to be more powerful than the original Bonferroni procedure under the condition of controlling overall Type I error for a given α level. It is interesting to find out how much more power can be gained by using a modified procedure, and among the modified Bonferroni procedures, which one is most powerful.

In this section, the original Bonferroni procedure is compared with four of the modified Bonferroni techniques. The focus of the comparison is to determine the practical differences in the number of hypotheses rejected by each procedure using real educational and psychological research data. We realize that the increased power by newer procedures may not necessarily translate into a substantive increase in the number of hypotheses rejected with real data sets. The Rom (1990) procedure is excluded from this comparison, because as the number of the tests increases, the calculations become very complicated and it is prohibitive for practical applications even when using a computer.

Source of Data

A total of 50 correlation matrices was selected from four reputable journals in education and psychology (American Educational Research Journal, 1981-1987; Educational and Psychological Measurement, 1985-1987; Journal of Vocational Behavior, 1984-1988; and Psychological Reports, 1986-1988). Each of the 50 matrices came from a different study. Only matrices for which sample sizes were reported are included in this study. The number of variables for each matrix ranges from seven to sixteen, the sample sizes range from 30 to 2944, and the number of multiple tests for each matrix ranges from 21 (for the 7x7 matrix) to 120 (for the 16x16 matrix) (Barton & Huberty, 1989). Across all matrices, a total of 2587 p values are obtained. A frequency distribution of these p values is presented in Table 9. Within each of the matrix, the percents of p values smaller than a given α level (.05, .10, and .20) are computed, the frequency distributions of these percents is presented in Table 10.

Insert Table 9 and 10 about here

Procedures

A SAS computer program was written for the data analysis. The program includes the following modules: (1) A p value corresponding to each correlation coefficient is computed as follows: First, the formula provided in SAS CORR procedure, $t=r[(n-2)^{1/2}]/[(1-r^2)^{1/2}]$, is used. Then the corresponding two-sided p value is determined by applying the SAS PROBT function: p=2{1-PROBT[ABS(t),DF]}. (2) All computed p values in a correlation matrix are ordered; (3) The original Bonferroni procedure and the four modified Bonferroni procedures are applied to each of the correlation matrices, using an overall α per matrix of .05, .10, and .20. On the computer printout, tables of results are displayed which have r values, corresponding p values, and criterion values for rejection using each of the five procedures being considered. [This program is available from the senior author.]

Results

Across the 50 correlation matrices, a total of 2587 hypotheses were tested. Tables 11 summarize the number of the rejections for the 2587 tests. All modified Bonferroni procedures reject more hypotheses than the original Bonferroni procedure. Among the four modified procedures, the Holm procedure has the lowest number of rejections and the Hommel procedure has the highest number of rejections whenever differences exist among the procedures. Overall, the SU procedures seem to be a little more powerful than the SD procedures. Within SD procedures, whenever the differences occurred, the Holland-Copenhaver procedure has a slightly higher number of rejections than the Holm procedure. Within SU procedures, whenever the differences occurred, the Hommel procedure has a slightly higher number of rejections than the Hochberg procedure.

Insert Table 11 about here

In general, for a relatively large number of matrices, the modified Bonferroni procedures rejected the same number of hypotheses as the original Bonferroni procedure. For the 50 correlation matrices, Table 12 displays the number of matrices which have the same number of rejections across all methods, the number of the matrices which had differences between the original Bonferroni procedure (B) and the modified Bonferroni procedures (MBs), the number of the matrices for which the modified Bonferroni procedures rejected the same number of hypotheses, and the number of the matrices which had different number of rejections when using different modified Bonferroni procedures. For most of the situations where there are differences between original Bonferroni procedure and modified Bonferroni procedures, the differences among the four modified Bonferroni procedures are quite small.

Insert Table 12 about here

From our sample of matrices, it appears that the higher the overall Type I error rate, the more differences are observed across all the procedures. When the overall α is .05, there are few differences in the number of rejections. When the overall α is .20, the differences among the methods increases in terms of number of rejections.

The 50 matrices were also ordered by the number of multiple tests and the sample size. No obvious patterns of differences were discovered in terms of relationships among the number of rejections by the methods, the number of multiple tests, and sample size. The correlations of the number of rejections among the five procedures are all greater than .995. The correlations between the number of rejections and the number of tests across the five procedures are very nearly the same (from .61 to .63); similarly for the correlations between the number of rejections and sample size (from .42 to .44).

Discussion

Holland and Copenhaver (1988) recommended that a modified Bonferroni procedure be used in situations where the original Bonferroni would otherwise be the method of choice. The results of the current study do not totally support that conclusion; the original Bonferroni procedure appears to be only slightly less powerful. Although the modified Bonferroni approaches reject more individual hypotheses, the percent of gain is less than 4% across 2587 hypotheses tested. Among the modified procedures, the differences in the number of rejections are even smaller.

Hommel (1989) analytically proved that the Hommel procedure is at least as powerful as the Hochberg procedure, and in general more powerful. The results from the present study are consistant with this finding. Dunnett and Tamhane (1992) found power

increases yielded by the Hommel and the Rom procedures over the Hochberg procedure are marginal at the best, with the Rom procedure being slightly superior. The results of current study partially supported their finding by showing that the Hommel procedure had slightly more rejections. Hochberg and Benjamini (1990) compared the Holm procedure and the Hochberg procedure with two sets of multiple comparisons data and failed to detect any difference in the number of rejections.

However, for a set of 12 tests presented by Holland and Copenhaver (1988), we found that the Hochberg procedure rejects six more hypotheses than the Holm procedure. Similar situations are observed for a few matrices in the present study. It appears that for the situations where some obtained p values are very close together and either are not very small or are not very large, the modified Bonferroni procedures are potentially more powerful, and the newly developed SU approaches are likely to be more powerful than SD approaches.

Finally, Table 13 gives critical values for five of the procedures discussed in this paper. Critical values for the Hommel procedure depend on obtained p values, so they can not be included in this table. The table is suitable for the situations where the number of the multiple tests ranges from 2 to 10 and the overall α is .05, .10, or .20. Looking at the critical values from all the modified procedures, it is obvious that unless the distribution of a set of p values is very special, the results from all the procedures are likely to be very similar.

Insert Table 13 about here

The results and the discussions of this study are only based on a set of 50 correlation matrices. For the further investigations, it will be interesting to compare the modified Bonferroni procedures in other multiple testing contexts with real data as well as with simulated data.

There is another potential interesting problem that may be addressed in the multiple test arena. This problem pertains to the interrelationship of the overall α, power, number of tests, effect size, and sample size. Is it reasonable to use an overall α level of .05 for the 45 tests based on a 10x10 correlation matrix (using any one of the modified Bonferroni procedures)? What sample size would one need (with some expected minimum average correlation) to obtain an average (across the 45 tests) power of, say, .70? [Other definitions of power may be considered; see Einot & Gabriel (1975) and Ramsey (1978)]. Perhaps sample-size tables could be constructed for which overall α, average (or overall) test power, number of tests, and average effect size are used as input.

REFERENCES

Barton, R. M., & Huberty, C. J. (1989, April). Multiple testing and correlation matrices: An application of three procedures. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Collis, B. A., & Rosenblood, L. K. (1985). The problem of inflated significance when testing individual correlations from a correlation matrix. Journal for Research in Mathematics Education, 16, 52-55.

Darlington, R. B. (1990). Regression and linear models. New York: McGraw-Hill.

Dunnett, C. W., & Tamhane, A. C. (1992). A step-up multiple test procedure. Journal of the American Statistical Association, 87, 162-170.

Einot, I., & Gabriel, K. R. (1975). A study of the powers of several methods of multiple comparisons. Journal of the American Statistical Association, 70, 574-583.

Fletcher, H. J., Daw, H., & Young, J. (1989). Controlling multiple F test errors with an overall F test. Applied Behavioral Science, 25, 101-108.

Hill, M. (1969). On looking at large correlation matrices. Biometrika, 56, 249-253.

Hochberg, Y. (1988). A sharper Bonferroni procedure for multiple tests of significance. Biometrika, 75, 800-802.

Hochberg, Y.,& Benjamini, Y. (1990). More powerful procedures for multiple significance testing. Statistics in Medicine, 9, 811-818.

Holland, B. (1991). Comment on Saville (letter to editor). The American Statistician, 45, 165.

Holland, B. S., & Copenhaver, M. D. (1987). An improved sequentially rejective Bonferroni test procedure. Biometrics, 43, 417-423.

Holland, B. S., & Copenhaver, M. D. (1988). Improved Bonferroni-type multiple testing procedures. Psychological Bulletin, 104, 145-149.

Holm, S. (1979). A simple sequentially rejective multiple test procedure. Scandinavian Journal of Statistics, 6, 65-70.

Hommel, G. (1988). A stagewise rejective multiple test procedure based on a modified Bonferroni test. Biometrika, 75, 383-386.

Hommel, G. (1989). A comparison of two modified Bonferroni procedures. Biometrika, 76,624-625.

Hsiung, T. H., & Olejnik, S. F. (1991, April). Power of pairwise multiple comparisons in the unequal variance case. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Keppel, G. (1991). Design and analysis: A researcher's handbook (3rd ed.). Englewood Cliffs, NJ: Prentice Hall.

Keppel, G., & Zedeck, S. (1989). Data analysis for research designs: Analysis of variance and multiple regression/correlation approaches. New York: Freeman.

Keselman, H. J., Keselman, J. C., & Games, P. A. (1991). Maximum familywise Type I error rate: The least significant difference, Newman-Keuls, and other multiple comparison procedures. Psychological Bulletin, 110, 155-161.

Keselman, H. J., Keselman, J. C., & Shaffer, J. P. (1991). Multiple pairwise comparisons of repeated measures means under violation of multisample sphericity. Psychological Bulletin, 110, 162-170.

Koziol, J. A. (1989). A note on plots of p-values to evaluate many tests simultaneously. Biometrical Journal, 8, 969-972.

Larzelere, R. E., & Mulaik, S. A. (1977). Single-sample tests for many correlations. Psychological Bulletin, 84, 557-569.

Lea, P. (1991). Multiple confusions (letter to editor). The American Statistician, 45, 165-166.

Looney, S. W., and Stanley, W. B. (1989). Exploratory repeated measures analysis for two or more groups: Review and update. The American Statistician, 43, 220-225.

Manly, B. F. J., & McAlevey, L. (1987). A randomization alternative to the Bonferroni inequality with multiple F tests. Proceedings of the Second International Tamperer Conference in Statistics, 567-573.

Maxwell S. E., & Delaney, H. D. (1990). Designing experiments and analyzing data: A Model comparison perspective. Belmont, CA: Wadsworth.

Ramsey, P. H. (1978). Power differences between pairwise multiple comparisons. Journal of the American Statistical Association, 73, 479-485.

Richardson, A. G. (1988). Classroom learning environment and creativity: Some caribbean findings. Psychological Reports, 62, 939-942.

Rom, D. M. (1990). A sequentially rejective test procedure based on a modified Bonferroni inequality. Biometrika, 77, 663-665.

Ryan, T. A. (1959). Multiple comparisons in psychological research. Psychological Bulletin, 56, 26-47.

Saville, D. J. (1990). Multiple comparison procedures: The practical solution. The American Statistician, 44, 174-180.

Seaman, M.A., Levin, J. R., & Serlin, R. C. (1991). New developments in pairwise multiple comparisons: Some powerful and practicable procedure. Psychological Bulletin, 110, 577-586.

Seaman, M.A., Levin, J. R., Serlin, R. C., & Franke, T. M. (1990, April). A modified algorithm and an examination of power for Peritz's pairwise multiple-comparison procedure. Paper presented at the annual meeting of American Educational Research Association, Boston, MA.

Shaffer, J. P. (1986). Modified sequentially rejective multiple test procedures. Journal of the American Statistical Association, 81, 826-831.

Simes, R. J. (1986). An improved Bonferroni procedure for multiple tests of significance. Biometrika, 73, 751-754.

Stevens, J. (1986). Applied multivariate statistics for the social sciences. Hillsdale, NJ: Erlbaum.

Stevens, J. (1990). Intermediate statistics: A modern approach. Hillsdale, NJ: Erlbaum.

Toothaker, L. E. (1991). Multiple comparisons for researchers. Newbury, CA: Sage.

Worsley, K. J. (1982). An improved Bonferroni inequality and applications. Biometrika, 69, 297-302.

Note: The reference list for the studies providing the 50 correlation matrices is available upon request.

Table 1

Correlation Coefficients and Associated p values

	V2	V3	V4	V5
V1	-.41 .0000	-.15 .0633	.17 .0350	.15 .0633
V2		.26 .0011	-.21 .0089	.18 .0255
V3			-.09 .2670	-.20 .0129
V4				.31 .0001

Note: This is a partial correlation matrix from the study conducted by Richardson (1988).

Table 2

Ordered r and p values

r	$p_{(i)}$	$H_{(i)}$
-.41	.0000	$H_{(1)}$
.31	.0001	$H_{(2)}$
.26	.0011	$H_{(3)}$
-.21	.0089	$H_{(4)}$
-.20	.0129	$H_{(5)}$
-.18	.0255	$H_{(6)}$
.17	.0350	$H_{(7)}$
-.15	.0633	$H_{(8)}$
.15	.0633	$H_{(9)}$
-.09	.2670	$H_{(10)}$

Table 3

Illustrations for Holm (1979) Procedure

Hypothesis Tested	Obtained $p_{(i)}$	i	$\alpha/(m-i+1)$	$\alpha_{(i)}'$	Results
$H_{(1)}$	.0000	1	.10/(10-1+1)	.0100	$p_{(1)} < .0100$ *
$H_{(2)}$	.0001	2	.10/(10-2+1)	.0111	$p_{(2)} < .0111$ *
$H_{(3)}$	.0011	3	.10/(10-3+1)	.0125	$p_{(3)} < .0125$ *
$H_{(4)}$	.0089	4	.10/(10-4+1)	.0143	$p_{(4)} < .0143$ *
$H_{(5)}$	.0129	5	.10/(10-5+1)	.0167	$p_{(5)} < .0167$ *
$H_{(6)}$	.0255	6	.10/(10-6+1)	.0200	$p_{(6)} > .0200$

* $H_{(i)}$ is rejected.

Table 4

Illustrations for Holland and Copenhaver (1987) Procedure

Hypothesis Tested	Obtained $p_{(i)}$	i	$1-(1-\alpha)^{1/(m-i+1)}$	$\alpha_{(i)}'$	Results
$H_{(1)}$	.0000	1	$1-(1-.10)^{1/(10-1+1)}$	.0105	$p_{(1)} < .0105$ *
$H_{(2)}$	.0001	2	$1-(1-.10)^{1/(10-2+1)}$	.0116	$p_{(2)} < .0116$ *
$H_{(3)}$	.0011	3	$1-(1-.10)^{1/(10-3+1)}$	.0131	$p_{(3)} < .0131$ *
$H_{(4)}$	.0089	4	$1-(1-.10)^{1/(10-4+1)}$	.0149	$p_{(4)} < .0149$ *
$H_{(5)}$	.0129	5	$1-(1-.10)^{1/(10-5+1)}$	.0174	$p_{(5)} < .0174$ *
$H_{(6)}$	.0255	6	$1-(1-.10)^{1/(10-6+1)}$	.0209	$p_{(6)} > .0209$

* $H_{(i)}$ is rejected.

Table 5

Illustrations for Hochberg (1988) Procedure

$H_{(i)}$ Tested	Obtained $p_{(i)}$	i	$\alpha/(m-i+1)$	$\alpha_{(i)}'$	Results
$H_{(10)}$	.2670	10	.10/(10-10+1)	.1000	$p_{(10)} > .1000$
$H_{(9)}$	.0633	9	.10/(10-9+1)	.0500	$p_{(9)} > .0500$
$H_{(8)}$	.0633	8	.10/(10-8+1)	.0333	$p_{(8)} > .0333$
$H_{(7)}$	.0350	7	.10/(10-7+1)	.0250	$p_{(7)} > .0250$
$H_{(6)}$	.0255	6	.10/(10-6+1)	.0200	$p_{(6)} > .0200$
$H_{(5)}$	.0129	5	.10/(10-5+1)	.0167	$p_{(5)} < .0167$ *

* $H_{(6)}$ is rejected.

Table 6

Illustrations for Hommel (1988) Procedure (First Stage)

Step i'	Test k	Obtained $p_{(m-i'+k)} = p_{(i)}$	$k\alpha/i'$	$p_{(i)} > k\alpha/i'$	j
1	1	$p_{(10-1+1)} = p_{(10)} = .2670$	1(.10)/1=.1000	Yes	1
2	1	$p_{(10-2+1)} = p_{(9)} = .0633$	1(.10)/2=.0500	Yes	
	2	$p_{(10-2+2)} = p_{(10)} = .2670$	2(.10)/2=.1000	Yes	2
3	1	$p_{(10-3+1)} = p_{(8)} = .0633$	1(.10)/3=.0333	Yes	
	2	$p_{(10-3+2)} = p_{(9)} = .0633$	2(.10)/3=.0677	No	
The first stage stops here. We have j=2.					

Table 7

Illustrations for Rom (1990) Procedure

Hypo. Tested	Obtained $p_{(i)}$	Computing Equation	$\alpha_{(i)}'$	Results
$H_{(1)}$	.2670	$\alpha_{(1)}'=\alpha$	.1000	$p_{(1)} > \alpha_{(1)}'$
$H_{(2)}$	.0633	$\alpha_{(2)}'=\alpha_{(1)}'/2$	.0500	$p_{(2)} > \alpha_{(2)}'$
$H_{(3)}$	.0633	$\alpha_{(3)}'=[\alpha^1+\alpha^2-3(\alpha_{(2)}'^2)]/3$	.0342	$p_{(3)} > \alpha_{(3)}'$
$H_{(4)}$	.0350	$\alpha_{(4)}'=[\alpha^1+\alpha^2+\alpha^3-4(\alpha_{(2)}'^3)-6(\alpha_{(3)}'^2)]/4$	.0259	$p_{(4)} > \alpha_{(4)}'$
$H_{(5)}$	.0255	$\alpha_{(5)}'=[\alpha^1+\alpha^2+\alpha^3+\alpha^4-5(\alpha_{(2)}'^4)-10(\alpha_{(3)}'^3)-10(\alpha_{(4)}'^2)]/5$	.0208	$p_{(5)} > \alpha_{(5)}'$
$H_{(6)}$	.0129	$C_{(6)}'=[\alpha^1+\alpha^2+\alpha^3+\alpha^4+\alpha^5-6(\alpha_{(2)}'^5)-15(\alpha_{(3)}'^4)-20(\alpha_{(4)}'^3)-15(\alpha_{(5)}'^2)]/6$	.0174	$p_{(6)} < \alpha_{(6)}'$ *

* $H_{(6)}$ is rejected.

Table 8

Number of Rejections at Different α Levels with the Example Data

α	Bonferroni	Holm (SS)	Holland Copenhaver (SD)	Hochberg (SU)	Hommel (SU)	Rom (SU)
.05	3	3	3	3	3	3
.10	4	5	5	5	7	5
.20	5	9	9	9	9	9

Table 9

Frequencies of p values from all 50 Correlation Matrices

p value	Frequency	Percent
$\leq$ .0010	1275	49.3
.0011 - .0050	144	5.6
.0051 - .0100	67	2.6
.0101 - .0500	208	8.0
.0501 - .1000	123	4.8
.1001 - .1500	93	3.6
.1501 - .2000	75	2.9
> .2001	602	23.3
	2587	100.1

Table 10

Grouped Frequency Distribution of the 50 Correlation Matrices with the Percent of p values within the Given Intervals at Three α Levels

% of p values less than α	α=.05	α=.10	α=.20
11-20	1	1	0
21-30	5	1	1
31-40	4	6	1
41-50	4	2	3
51-60	6	6	6
61-70	4	**6**	6
71-80	7	3	7
81-90	4	9	7
91-100	15	16	19
	50	50	50

Note: The bold-faced **6** indicates that six out of 50 matrices had 61-70 percent of p values that were less than .10.

Table 11

Total Numbers of Rejections with Each Procedure across 50 Matrices

α	Bonferroni (SS)	Holm (SD)	Holland Copenhaver (SD)	Hochberg (SU)	Hommel (SU)
.05	1270 (49.1)	1343 (51.9)	1343 (51.9)	1349 (52.2)	1353 (52.3)
.10	1323 (51.1)	1410 (54.5)	1413 (54.6)	1417 (54.8)	1420 (54.9)
.20	1395 (53.9)	1473 (56.9)	1485 (57.4)	1484 (57.4)	1496 (57.8)

Note: The numbers in the parentheses are the percents.

Table 12

Number of Matrices out of 50 which had Differences in the Number of Rejections among the Procedures

α	No Diff. for All	Diff. Between B and MBs	No Diff. for All MBs	Diff. Among Four MBs
.05	25	25	19	6
.10	20	30	25	5
.20	15	35	20	15

Table 13

Adjusted alpha values for selected Bonferroni-type procedures for overall alpha equalling .05.

	m=2					m=3					m=4					m=5					m=6				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.0250	.0250	.0253	.0250	.0250	.0167	.0167	.0170	.0167	.0169	.0125	.0125	.0127	.0125	.0127	.0100	.0100	.0102	.0100	.0102	.0083	.0083	.0085	.0083	.0085
2	.0250	.0500	.0500	.0500	.0500	.0167	.0250	.0253	.0250	.0250	.0125	.0167	.0170	.0167	.0169	.0100	.0125	.0127	.0125	.0127	.0083	.0100	.0102	.0100	.0102
3						.0167	.0500	.0500	.0500	.0500	.0125	.0250	.0253	.0250	.0250	.0100	.0167	.0170	.0167	.0169	.0083	.0125	.0127	.0125	.0127
4											.0125	.0500	.0500	.0500	.0500	.0100	.0250	.0253	.0250	.0250	.0083	.0167	.0170	.0167	.0169
5																.0100	.0500	.0500	.0500	.0500	.0083	.0250	.0253	.0250	.0250
6																					.0083	.0500	.0500	.0500	.0500

	m=7					m=8					m=9					m=10				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.0071	.0071	.0073	.0071	.0073	.0062	.0062	.0064	.0062	.0064	.0056	.0056	.0056	.0056	.0057	.0050	.0050	.0051	.0050	.0051
2	.0071	.0083	.0085	.0083	.0085	.0062	.0071	.0073	.0071	.0073	.0056	.0062	.0064	.0062	.0064	.0050	.0056	.0057	.0056	.0057
3	.0071	.0100	.0102	.0100	.0102	.0062	.0083	.0085	.0083	.0085	.0056	.0071	.0073	.0071	.0073	.0050	.0062	.0064	.0062	.0064
4	.0071	.0125	.0127	.0125	.0127	.0062	.0100	.0102	.0010	.0102	.0056	.0083	.0085	.0083	.0085	.0050	.0071	.0073	.0071	.0073
5	.0071	.0167	.0170	.0167	.0169	.0062	.0125	.0127	.0125	.0127	.0056	.0100	.0102	.0100	.0102	.0050	.0083	.0085	.0083	.0085
6	.0071	.0250	.0253	.0250	.0250	.0062	.0167	.0170	.0167	.0170	.0056	.0125	.0127	.0125	.0127	.0050	.0100	.0102	.0100	.0102
7	.0071	.0500	.0500	.0500	.0500	.0062	.0250	.0253	.0250	.0250	.0056	.0167	.0170	.0167	.0169	.0050	.0125	.0127	.0125	.0127
8						.0062	.0500	.0500	.0500	.0500	.0056	.0250	.0253	.0250	.0250	.0050	.0167	.0170	.0167	.0169
9											.0056	.0500	.0500	.0500	.0500	.0050	.0250	.0253	.0250	.0250
10																.0050	.0500	.0500	.0500	.0500

Note: B=Bonferroni(SS), H=Holm(SD), HC=Holland and Copenhaver(SD), HB=Hochberg(SU), and R=Rom(SU).

BEST COPY AVAILABLE

Table 13 (continued)

Adjusted alpha values for selected Bonferroni-type procedures for overall alpha equalling .10.

	m=2					m=3					m=4					m=5					m=6				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.0500	.0500	.0513	.0500	.0500	.0333	.0333	.0345	.0333	.0342	.0250	.0250	.0260	.0250	.0259	.0200	.0200	.0209	.0200	.0208	.0167	.0167	.0174	.0167	.0174
2	.0500	.1000	.1000	.1000	.1000	.0333	.0500	.0513	.0500	.0500	.0250	.0333	.0345	.0333	.0342	.0200	.0250	.0260	.0250	.0259	.0167	.0200	.0209	.0200	.0208
3						.0333	.1000	.1000	.1000	.1000	.0250	.0500	.0513	.0500	.0500	.0200	.0333	.0345	.0333	.0342	.0167	.0250	.0260	.0250	.0259
4											.0250	.1000	.1000	.1000	.1000	.0200	.0500	.0513	.0500	.0500	.0167	.0333	.0345	.0333	.0342
5																.0200	.1000	.1000	.1000	.1000	.0167	.0500	.0513	.0500	.0500
6																					.0167	.1000	.1000	.1000	.1000

	m=7					m=8					m=9					m=10				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.0143	.0143	.0149	.0143	.0149	.0125	.0125	.0131	.0125	.0131	.0111	.0111	.0116	.0111	.0116	.0100	.0100	.0105	.0100	.0105
2	.0143	.0167	.0174	.0167	.0174	.0125	.0143	.0149	.0143	.0149	.0111	.0125	.0131	.0125	.0131	.0100	.0111	.0116	.0111	.0116
3	.0143	.0200	.0209	.0200	.0208	.0125	.0167	.0174	.0167	.0174	.0111	.0143	.0149	.0143	.0149	.0100	.0125	.0131	.0125	.0131
4	.0143	.0250	.0260	.0250	.0259	.0125	.0200	.0209	.0200	.0208	.0111	.0167	.0174	.0167	.0174	.0100	.0143	.0149	.0143	.0149
5	.0143	.0333	.0345	.0333	.0342	.0125	.0250	.0260	.0250	.0259	.0111	.0200	.0209	.0200	.0208	.0100	.0167	.0174	.0167	.0174
6	.0143	.0500	.0513	.0500	.0500	.0125	.0333	.0345	.0333	.0342	.0111	.0250	.0260	.0250	.0259	.0100	.0200	.0209	.0200	.0208
7	.0143	.1000	.1000	.1000	.1000	.0125	.0500	.0513	.0500	.0500	.0111	.0333	.0345	.0333	.0342	.0100	.0250	.0260	.0250	.0259
8						.0125	.1000	.1000	.1000	.1000	.0111	.0500	.0513	.0500	.0500	.0100	.0333	.0345	.0333	.0342
9											.0111	.1000	.1000	.1000	.1000	.0100	.0500	.0513	.0500	.0500
10																.0100	.1000	.1000	.1000	.1000

Note: B=Bonferroni(SS), H=Holm(SD), HC=Holland and Copenhaver(SD), HB=Hochberg(SU), and R=Rom(SU)

BEST COPY AVAILABLE

Table 13 (continued)

Adjusted alpha values for selected Bonferroni-type procedures for overall alpha equalling .20.

	m=2					m=3					m=4					m=5					m=6				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.1000	.1000	.1056	.1000	.1000	.0667	.0667	.0717	.0667	.0700	.0500	.0500	.0543	.0500	.0537	.0400	.0400	.0436	.0400	.0434	.0333	.0333	.0365	.0333	.0364
2	.1000	.2000	.2000	.2000	.2000	.0667	.1000	.1056	.1000	.1000	.0500	.0667	.0717	.0667	.0700	.0400	.0500	.0543	.0500	.0537	.0333	.4000	.0436	.0400	.0434
3						.0667	.2000	.2000	.2000	.2000	.0500	.1000	.1056	.1000	.1000	.0400	.0667	.0717	.0667	.0700	.0333	.0500	.0543	.0500	.0537
4											.0500	.2000	.2000	.2000	.2000	.0400	.1000	.1056	.1000	.1000	.0333	.0667	.0717	.0667	.0700
5																.0400	.2000	.2000	.2000	.2000	.0333	.1000	.1056	.1000	.1000
6																					.0333	.2000	.2000	.2000	.2000

	m=7					m=8					m=9					m=10				
i	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R	B	H	HC	HB	R
1	.0286	.0286	.0314	.0286	.0313	.0250	.0250	.0275	.0250	.0275	.0222	.0222	.0245	.0222	.0245	.0200	.0200	.0221	.0200	.0220
2	.0286	.0333	.0365	.0333	.0364	.0250	.0286	.0314	.0286	.0313	.0222	.0250	.0275	.0250	.0275	.0200	.0222	.0245	.0222	.0245
3	.0286	.0400	.0436	.0400	.0434	.0250	.0333	.0365	.0333	.0364	.0222	.0286	.0314	.0286	.0313	.0200	.0250	.0275	.0250	.0275
4	.0286	.0500	.0543	.0500	.0537	.0250	.0400	.4369	.0400	.0434	.0222	.0333	.0365	.0333	.0364	.0200	.0286	.0314	.0286	.0313
5	.0286	.0667	.0717	.0667	.0700	.0250	.0500	.0543	.0500	.0537	.0222	.0400	.0436	.0400	.0434	.0200	.0333	.0365	.0333	.0364
6	.0286	.1000	.1056	.1000	.1000	.0250	.0667	.0717	.0667	.0700	.0222	.0500	.0543	.0500	.0537	.0200	.0400	.0436	.0400	.0434
7	.0286	.2000	.2000	.2000	.2000	.0250	.1000	.1056	.1000	.1000	.0222	.0667	.0717	.0667	.0700	.0200	.0500	.0543	.0500	.0537
8						.0250	.2000	.2000	.2000	.2000	.0222	.1000	.1056	.1000	.1000	.0200	.0667	.0717	.0667	.0700
9											.0222	.2000	.2000	.2000	.2000	.0200	.1000	.1056	.1000	.1000
10																.0200	.2000	.2000	.2000	.2000

Note: B=Bonferroni(SS), H=Holm(SD), HC=Holland and Copenhaver(SD), HB=Hochberg(SU), and R=Rom(SU).

BEST COPY AVAILABLE